

Grade 11-12 Math (HL) Curriculum

Unit of Study (Core Topic 1 of 7): Algebra

Curriculum Standards	Learner Outcomes
Sequences and Series	<ul style="list-style-type: none">• Uses arithmetic and geometric formulas.• Understands applications.• Recognizes infinite geometric series and finds their s.um.• Uses Sigma notation
Exponents and Logarithms	<ul style="list-style-type: none">• Knows laws of exponents and logarithms.• Changes bases.
Counting Principles	<ul style="list-style-type: none">• Uses permutations and combinations and know when to apply them
Binomial Theorem	<ul style="list-style-type: none">• Expands powers of binomials• Recognizes the Pascal triangle pattern
Mathematical Induction	<ul style="list-style-type: none">• Forms conjectures and prove them by mathematical induction
Complex Numbers	<ul style="list-style-type: none">• Understands the number i and the following terms: real part, imaginary part, conjugate, modulus, and argument• Manipulates Cartesian form and modulus-argument form• Applies the former to the complex plane (Argand diagram)• Performs sums, products and quotients of complex numbers• Uses De Moivre's Theorem and can calculate powers and roots of complex numbers• Finds conjugate roots of polynomial equations with real coefficients

Unit of Study: (Topic Core 2 of 7): Functions and Equations

Curriculum Standards	Learner Outcomes
Functions	<ul style="list-style-type: none">• Understands domain, range, and mapping notation• Works with composite functions and the identity function• Finds the inverse of given functions and their graphs if it exists
Graphing Functions	<ul style="list-style-type: none">• Graphs and analyzes functions on a GDC, investigating key features• Identifies asymptotes (horizontal and vertical)• Solves equations graphically, via algebra and/or GDC• Transforms graphs: translations, stretches, reflections, reciprocals, and absolute values• Recognizes the self-inverse nature of certain functions (eg. the reciprocal function)
Quadratic Functions	<ul style="list-style-type: none">• Recognizes standard vertex and factored forms• Finds the axis of symmetry, x intercepts, y intercepts.• Solves any quadratic equation (via factoring, quadratic formula, optional-completing the square)• Understands the discriminant

Exponential Functions	<ul style="list-style-type: none"> • Recognizes exponential equations and their applications • Recognizes logarithmic functions and their applications • Understands the inverse nature of exponents and logarithms
Inequalities	<ul style="list-style-type: none"> • Solves $g(x) > f(x)$ where f and g are linear or quadratic • Uses absolute value signs in inequalities
Advanced Polynomial Functions	<ul style="list-style-type: none"> • Uses the factor and remainder theorems • Applies the above to the solution of polynomial equations and inequalities • Understands the graphical significance of repeated roots

Unit of Study: (Core Topic 3 of 7): Circular Functions and Trigonometry

Curriculum Standards	Learner Outcomes
Circles	<ul style="list-style-type: none"> • Uses radians • Calculates lengths and areas of arcs and sectors
Unit Circle	<ul style="list-style-type: none"> • Defines cosine, sine, tangent, secant, cosecant, and cotangent in terms of the unit circle • Uses of the three Pythagorean Identities in Trigonometry • Recognizes tangent as slope of a line
Compound Angle Identities	<ul style="list-style-type: none"> • Uses the sum and difference formulae for sine, cosine, and tangent • Proves double angle identities
Double Angle Formulae	<ul style="list-style-type: none"> • Simplifies, solves and manipulates $\sin 2x$ and $\cos 2x$
Circular Functions	<ul style="list-style-type: none"> • Recognizes the periodic nature of trigonometric waves • Knows domains and ranges of all trig-waves • Works with the equation $f(x) = a \sin(b(x + c)) + d$ and identify their defining characteristics (eg. amplitude, period, translations) • Works with applications of said functions • Uses the three inverse functions arcsin, arccos and arctan and know their domains, ranges and graphs
Solving Trigonometric Equations over Finite Intervals	<ul style="list-style-type: none"> • Solves $a \sin(b(x + c)) = k$ by both analytical and graphical methods • Identifies quadratic equations inside trigonometric equations
Solving Triangles	<ul style="list-style-type: none"> • Uses the Cosine Rule and the Sine Rule • Recognizes the ambiguous case of the Sine Rule • Uses the area formula $K = .5ab \sin C$ • Applies the above to real life situations

Unit of Study: (Core Topic 4 of 7): Matrices

Curriculum Standards	Learner Outcomes
Matrix Language	<ul style="list-style-type: none">• Uses matrix terminology• Knows the term: element, row, column, and order
Algebra of Matrices	<ul style="list-style-type: none">• Performs addition, subtraction and multiplication by a scalar• Multiplies matrices by hand or by GDC• Recognizes identity and zero matrices
Square Matrices	<ul style="list-style-type: none">• Calculates determinants of 2×2 and 3×3 matrices• Calculates the inverse of a 2×2 matrix by hand and larger ones by GDC• Uses row reduction and augmented matrices
Systems of Linear Equations	<ul style="list-style-type: none">• Solves up to a maximum of three unknowns in three equations• Knows the conditions for the existence of unique solutions, no solutions and an infinity of solutions

Unit of Study: (Core Topic 5 or 7): Vectors

Curriculum Standards	Learner Outcomes
Vector Basics	<ul style="list-style-type: none">• Represents vectors in the plane and in three dimensions• Calculates distance between points in three dimensions• Writes vectors in component form and column representation (i, j and k as the standard basis for component form)• Calculates sums, differences, scalar multiplication and dot products of vectors• Finds direction and magnitude of any vector• Creates position vectors
Vector Relationships	<ul style="list-style-type: none">• Recognizes perpendicular and parallel cases• Calculates the angle between two vectors
Vector Equations	<ul style="list-style-type: none">• Represents applications in two and three dimensions by use of vector equations and their resulting parametric equations• Calculates the angle between two lines• Distinguish between coincident and parallel lines• Find intersections of lines by use of vector analysis and be aware of skew lines• Understands Cartesian form of lines in three dimensional space
Advanced Vector Mathematics	<ul style="list-style-type: none">• Uses the vector product and its determinant representation• Interprets the geometry of the vector product• Understands vector equations of planes and normal vectors• Understands Cartesian equations of planes• Finds intersections of: a line with a plane, two planes, and three planes• Calculates angles between: a line and a plane and two planes

Unit of Study: (Core Topic 6 of 7): Statistics and Probability

Curriculum Standards	Learner Outcomes
Statistical Basics	<ul style="list-style-type: none">• Understands the concepts of population, sample, random sample and frequency distribution of discrete and continuous data
Basic Statistical Graphs (Both continuous and discrete data)	<ul style="list-style-type: none">• Creates and reads the following: frequency tables, frequency diagrams, box and whisker plots, and frequency histograms• Calculates mid-interval values and interval width and upper/lower interval boundaries
Basic Statistical Measures	<ul style="list-style-type: none">• Calculates, means, medians, modes, quartiles, and percentiles• Calculates ranges, interquartile ranges, variances, and standard deviations• Is aware that the population mean is different than the sample mean and is generally unknown
Probability Basics	<ul style="list-style-type: none">• Understands concepts of trial, outcome, equally likely outcomes, sample space (U) and event• Calculates basic probabilities and their complements
Advanced Probability Formulas	<ul style="list-style-type: none">• Recognizes combined events and understand mutual exclusivity• Recognizes conditional probability and understands independence• Uses Venn diagrams, tree diagrams and tables of outcomes to solve problems• Uses Bayes' Theorem for two events
Probability Distributions	<ul style="list-style-type: none">• Creates and understands probability distributions• Finds expected value of assorted applications (eg. games of chance)• Defines and uses probability density functions
Binomial Distributions	<ul style="list-style-type: none">• Calculates entire binomial distributions and find their mean
Normal Distributions	<ul style="list-style-type: none">• Recognizes properties of normal distributions and z-scores• Standardizes normal variables and perform the reverse process using tables or a GDC
Poisson Distributions	<ul style="list-style-type: none">• Calculates entire Poisson distributions and find their mean

Unit of Study: (Core Topic 7 of 7): Calculus

Curriculum Standards	Learner Outcomes
The Derivative	<ul style="list-style-type: none">• Understands ideas of limit and convergence at a basic level• Finds derivative by first principles• Familiarizes with both forms of notation (dy/dx and $f'(x)$) for the first derivative• Finds the derivative to x^n, $\sin x$, $\tan x$, e^x, $\ln x$, a^x, $\log_a x$, $\arcsin x$, $\arccos x$, $\arctan x$• Interprets derivatives as slopes and rates of change• Finds equations of tangents and normals and identify increasing and decreasing functions
Advanced Differentiation	<ul style="list-style-type: none">• Derives sums and real multiples of the functions listed in the previous standard.• Performs the chain rule for composite functions• Applies the chain rule to related rates of change

	<ul style="list-style-type: none"> • Uses the product and quotient rules • Calculates the second derivative in both its notations
Graphical Interpretations	<ul style="list-style-type: none"> • Finds the local maximum and minimum points • Uses said points in optimization applications (eg. profit, area, volume...)
Indefinite Integration	<ul style="list-style-type: none"> • Understands integration as anti-differentiation • Finds the indefinite integral of x^n, $\sin x$, $\cos x$, $1/x$, and e^x • Integrates using basic substitution when dealing with linear composites
Definite Integration	<ul style="list-style-type: none"> • Can anti-differentiate with a boundary condition to determine the constant term • Evaluates definite integrals and can apply them to areas under and between curves • Calculates volumes of revolution about the x-axis and y-axis
Physics	<ul style="list-style-type: none"> • Solves kinematic involving displacement, velocity and acceleration • Recognizes area under velocity-time graphs as representation of distance
Graphical Behavior of Functions using Calculus	<ul style="list-style-type: none"> • Understands tangents, normals, horizontal and vertical and oblique asymptotes • Understanding significance of the second derivative (concavity) • Finds inflection points with zero and non-zero slopes
Implicit Differentiation	<ul style="list-style-type: none"> • Differentiates by indirect means

Curriculum Standards	Learner Outcomes
Further Integration	<ul style="list-style-type: none"> • Integrates by substitution and by parts • Changes limits in definite integrals
Basic Differential Equations	<ul style="list-style-type: none"> • Solves first order differential equations by separation of variables

Unit of Study: (Option Topic 8): Statistics and Probability

Curriculum Standards	Learner Outcomes
Expectation Algebra	<ul style="list-style-type: none"> • Performs linear transformations of single random variables • Finds means and variances of linear combinations of 2 independent random variables • Extends the above skill to n independent random variables
Advanced Distributions	<ul style="list-style-type: none"> • Works with discrete distributions (Bernoulli, binomial, negative binomial, Poisson, geometric, hypergeometric) in the solutions of probability mass functions and their means and variances • Works with continuous distributions (uniform, exponential, and normal) in the solutions of probability density functions and their means and variances • Distributes the sample mean and linear combinations of normal random variables • Uses the Central Limit Theorem • Estimates approximate normality for large samples
Confidence Intervals	<ul style="list-style-type: none"> • Finds confidence intervals for the mean of a population • Finds confidence intervals for the proportion of successes in a population

	<ul style="list-style-type: none">• Uses the t-distribution for unknown standard deviation regardless of sample size
Significance and Independence Testing	<ul style="list-style-type: none">• Performs significance tests for means and proportions• Finds/makes null and alternative hypotheses• Finds/interprets Type I and II errors• Understands critical regions, critical values, p-values, one/two-tailed tests• Performs the chi-squared test for frequency tables of varied degrees of freedom